Announcements

1) No more quizzes.

2) No more assignments, either!

3) Lutzer review online
   - win $25 Amazon gift card

4) Exam next Thursday will cover 3.7–3.9, 4.1–4.5, 5.1

Notation: (Integration shorthand)

To compute $\int_a^b f(x)\,dx$,

if h is any antiderivative of f,

you can shorthand

$$h(b)-h(a) \text{ as } h(x)\Big|_a^b .$$

Both are equal to $\int_a^b f(x)\,dx$.

Terminology (Indefinite Integrals)

- Section 4.4

If h is an antiderivative for f, we sometimes call h an indefinite integral of f, and write

$$h(x) = \int f(x)\,dx,$$

For example, if I write

$$\int (5x^4 + \sin(x))\, dx,$$

this means "find the general antiderivative of $5x^4 + \sin(x)$".

So $$\int (5x^4 + \sin(x))\, dx$$

$$= \boxed{x^5 - \cos(x) + C}$$

The Net Change Theorem

Section 4.4

Given a function f on an interval [a,b], if f is differentiable on an open interval containing [a,b], then

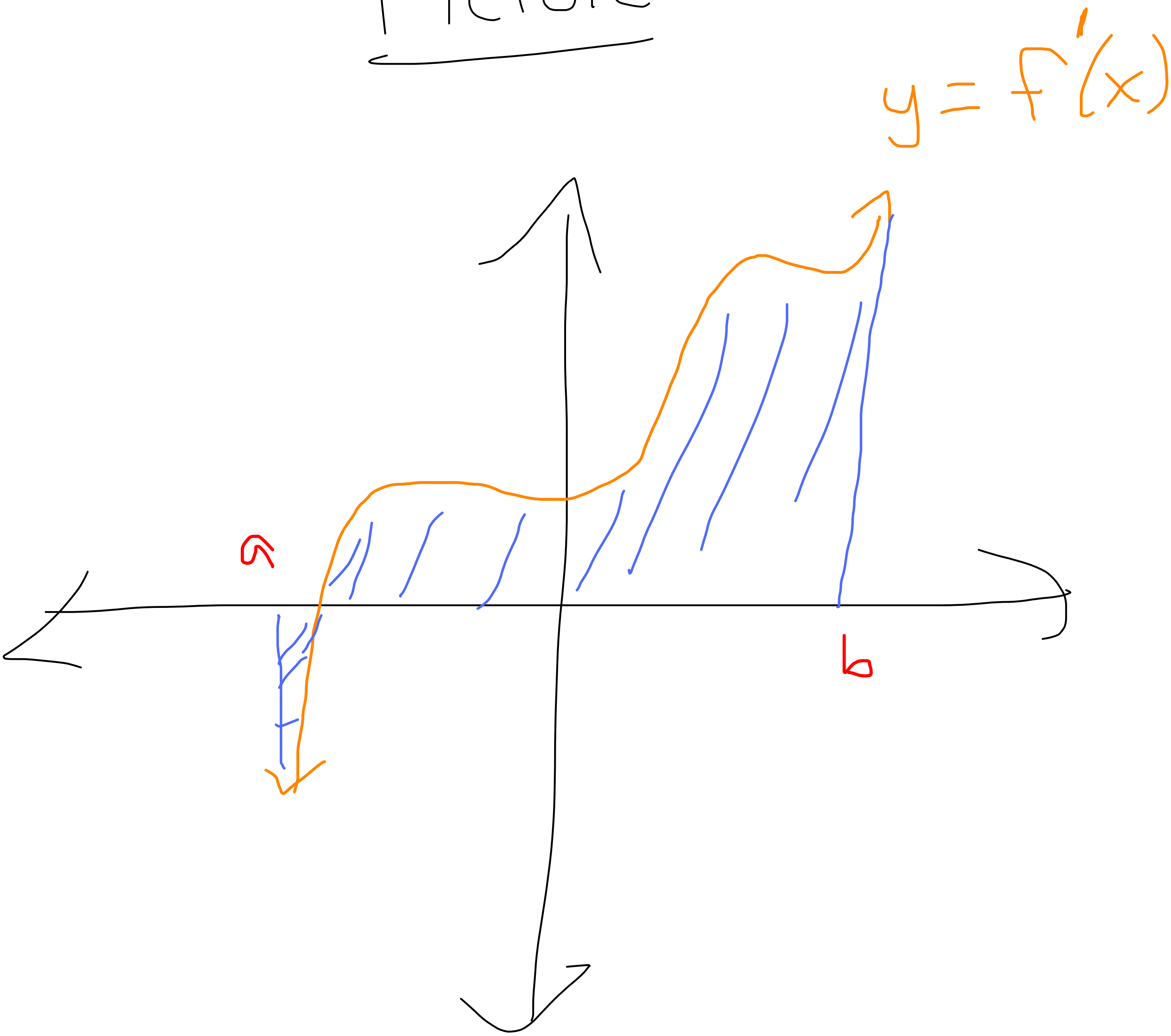
$$\boxed{f(b) - f(a) = \int_a^b f'(x)\,dx}$$

The net change of f on the interval [a,b] is $f(b)-f(a)$.

The theorem says that this net change is equal to

$$\int_a^b f'(x)\,dx.$$

Picture



By the fundamental theorem of calculus,

$$\int_a^b f'(x)\,dx = h(b) - h(a)$$

where h is any antiderivative of f'. In particular, we could choose $h = f$, to get

$$\int_a^b f'(x)\,dx = f(b) - f(a).$$

Example 1: If $E(t)$

= energy consumption in Detroit
(t = hours in a day)

Power $P(t) = E'(t)$

P measured in Megawatts,

E in megawatt-hours.

For Detroit

times

$$P(t) = 713777 \cdot 2\left(\cos(t) + \sqrt{t} + 7\right)$$

from 12 am to 12 pm

Regard 12 am as $t=0$.

We want the net change in energy from $t=0$ to $t=12$ (12 pm).

By the net change theorem, this is

$$E(12)-E(0)=\int_0^{12} E'(t)\,dt$$

$$=\int_0^{12} P(t)\,dt$$

Plugging in $P(t)$, this is

$$\int_0^{12} 1,427,554\left(\cos(t)+\sqrt{t}+7\right)dt$$

$$= 1,427,554 \int_0^{12} \left(\cos(t)+\sqrt{t}+7\right)dt$$

$$= 1,427,554\left(\sin(t)+\frac{2t^{3/2}}{3}+7t\right)\Bigg|_0^{12}$$

$$= \boxed{1,427,554\left(\sin(12)+\frac{2(12)^{3/2}}{3}+84\right)}$$

megawatt-hours

The Substitution Rule

Section 4.5

What happens if you can't just pick out an antiderivative?

$$\int x \sin(\pi x^2)\,dx$$

$$= \frac{-\cos(\pi x^2)}{2\pi} + C$$

why?

Rename variables — look for a function and its derivative somewhere inside the integral

$$\int x \sin(\pi x^2)\,dx$$

$$u = \pi x^2$$

$$\frac{du}{dx} = 2\pi x$$

so $x = \frac{1}{2\pi} \frac{du}{dx}$.

Substitute

$$\int x \sin(\pi x^2)\, dx$$

$$= \int \frac{1}{2\pi} \cdot \frac{du}{dx} \cdot \sin(u)\, dx$$

$$= \frac{1}{2\pi} \int \sin(u) \cdot \frac{du}{dx}\, dx$$

$$= \frac{1}{2\pi} \left(-\cos(u) + C\right)$$ (chain rule)

$$= \frac{1}{2\pi} \left(-\cos(\pi x^2) + C\right)$$

$$= \boxed{-\frac{\cos(\pi x^2)}{2\pi} + C}$$

Check that this is the answer by differentiating.

$$\frac{d}{dx}\left(\frac{-\cos(\pi x^2)}{2\pi}\right)$$

$$= \frac{1}{2\pi}\,\frac{d}{dx}\left(-\cos(\pi x^2)\right)$$

$$= \frac{1}{2\pi}\left(\overbrace{\sin(\pi x^2)\cdot\frac{d}{dx}(\pi x^2)}^{\text{chain rule}}\right)$$

$$= \frac{1}{\cancel{2\pi}}\left(\sin(\pi x^2)\cdot \cancel{2\pi} x\right.$$

$$= x\sin(\pi x^2)$$

This also works for definite integrals.

Theorem. (substitution) If u is continuous on $[a,b]$ and f is continuous on $u([a,b])$,

$$\boxed{\int_a^b f(u(x))\frac{du}{dx}\,dx = \int_{u(a)}^{u(b)} f(u)\,du}$$

Example 2:

$$\int_{\frac{\pi}{4}}^{5\pi/6} \sin^{5/2}(x)\cos(x)\,dx$$

$$= \int_{\frac{\pi}{4}}^{5\pi/6} \underbrace{(\sin(x))^{5/2}}_{u^{5/2}} \underbrace{\cos(x)\,dx}_{du}$$

$$u = \sin(x)$$

$$\frac{du}{dx} = \cos(x)$$

$$u\left(\frac{5\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$u\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

By substitution,

$$\int_{\frac{\pi}{4}}^{5\pi/6} \sin^{5/2}(x)\cos(x)\,dx$$

$$= \int_{\frac{\sqrt{2}}{2}}^{1/2} u^{5/2}\,du = \frac{2u^{7/2}}{7}\Bigg|_{\frac{\sqrt{2}}{2}}^{1/2}$$

$$= \boxed{\frac{2\left(1/2\right)^{7/2}}{7} - \frac{2\left(\frac{\sqrt{2}}{2}\right)^{7/2}}{7}}$$

Points: 1) You need $\frac{du}{dx}$ times f - plus, minus, or divided by is not enough.

2) Shorthand:

In previous problem, we had

$\frac{du}{dx} = \cos(x)$. Can write as

$du = \cos(x)dx$

3) You can either work problems as in the previous example or you can integrate indefinitely with respect to u, switch back to x, then plug in original bounds.

Warning: If you choose the first method, be sure to change your bounds!

Example 3: $\int \sqrt{9+x^4}\, x^{11}\, dx$

Think about this one, we'll do it Monday.